Autonomous and Adaptive Systems

Introduction to Deep Learning and Neural Architectures III

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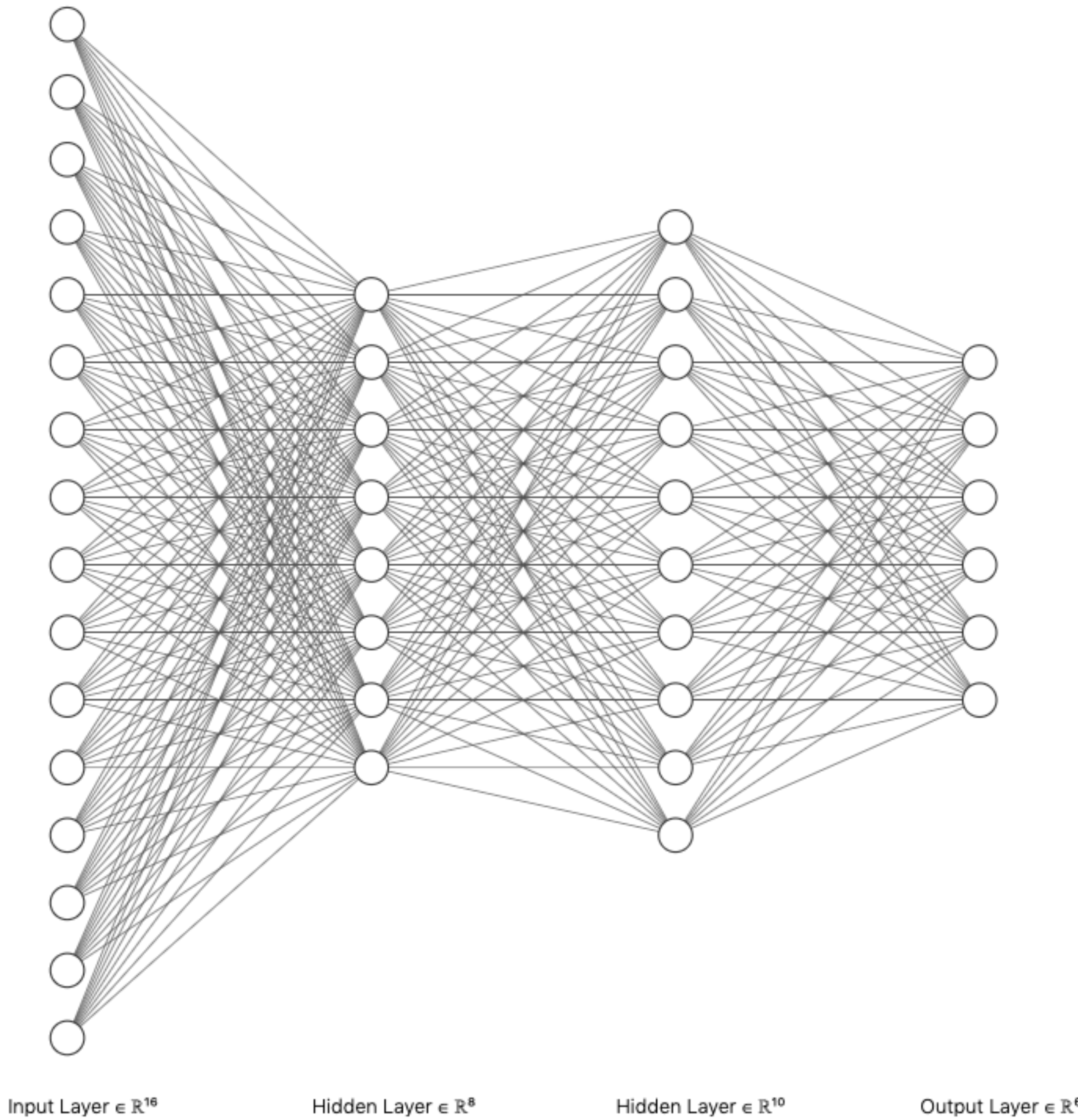
Deep Neural Networks



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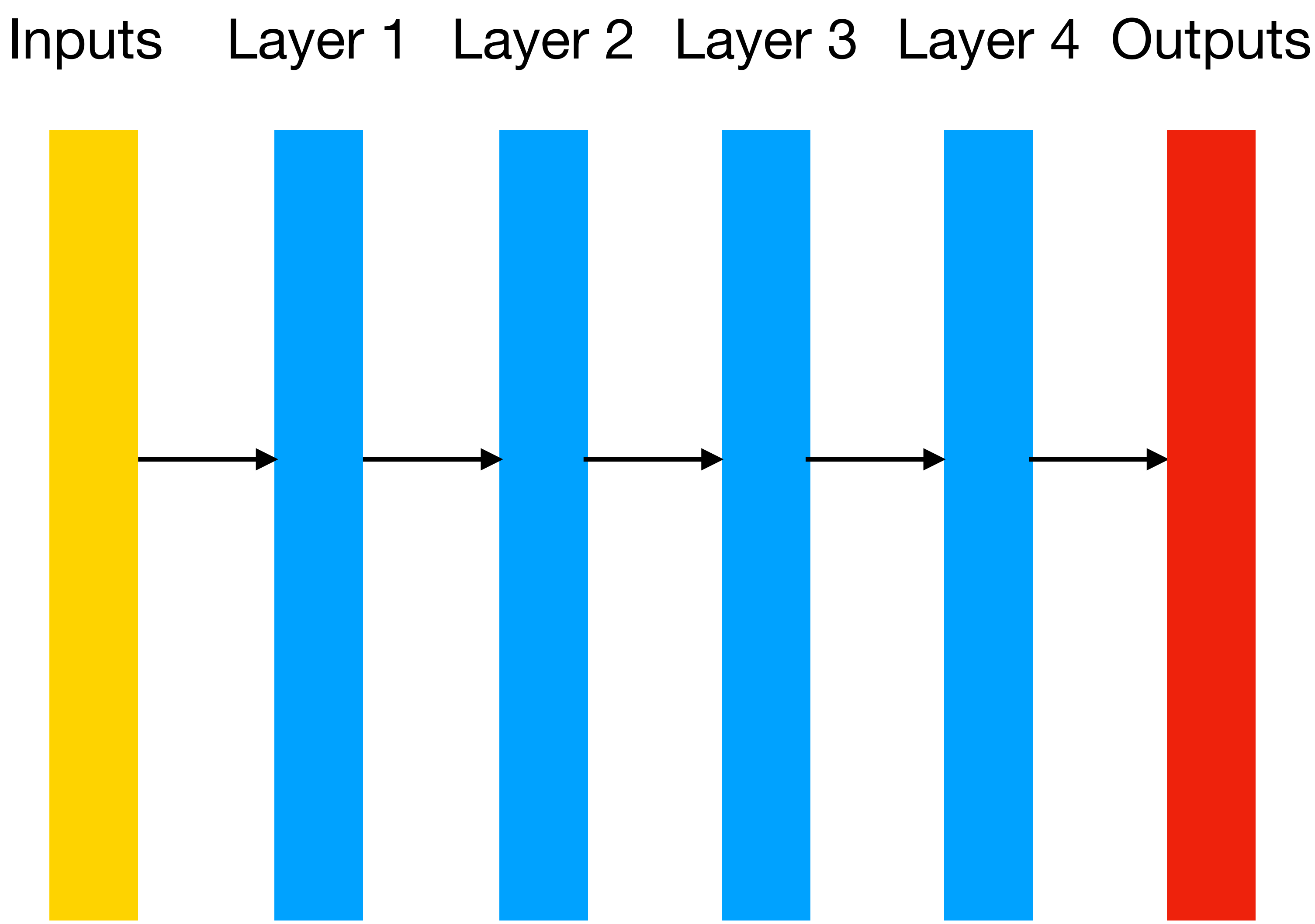
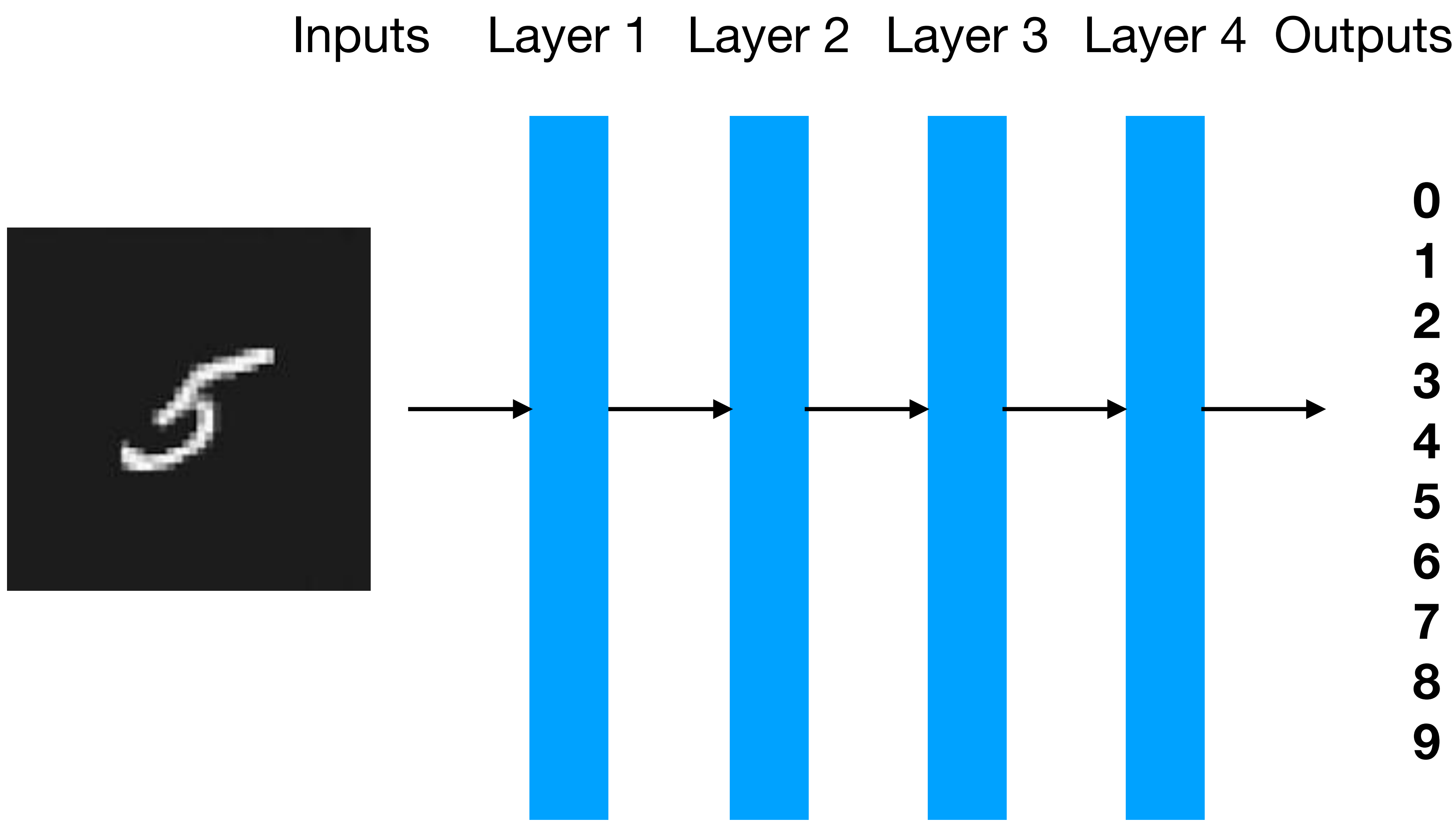
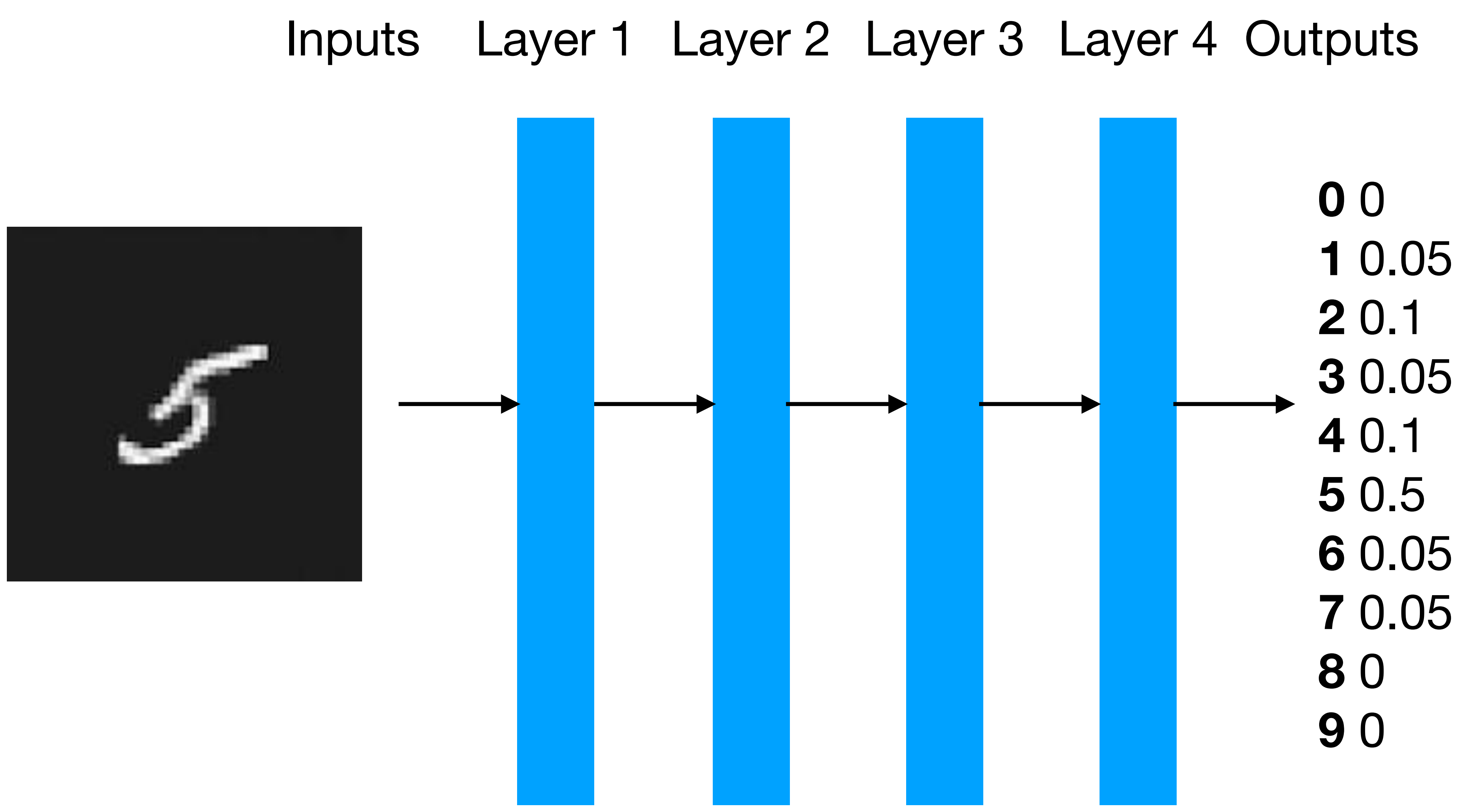


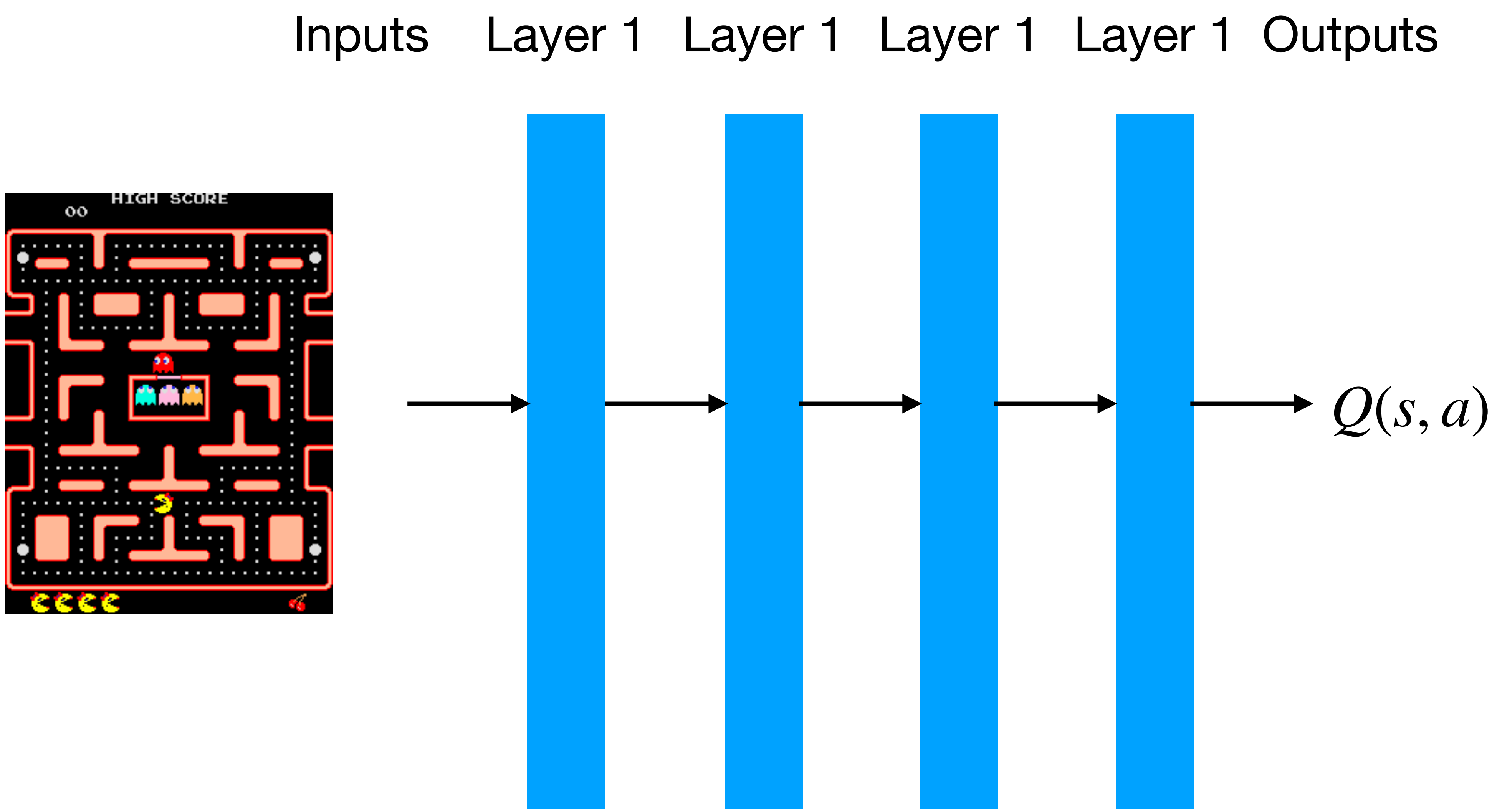
Deep Neural Networks



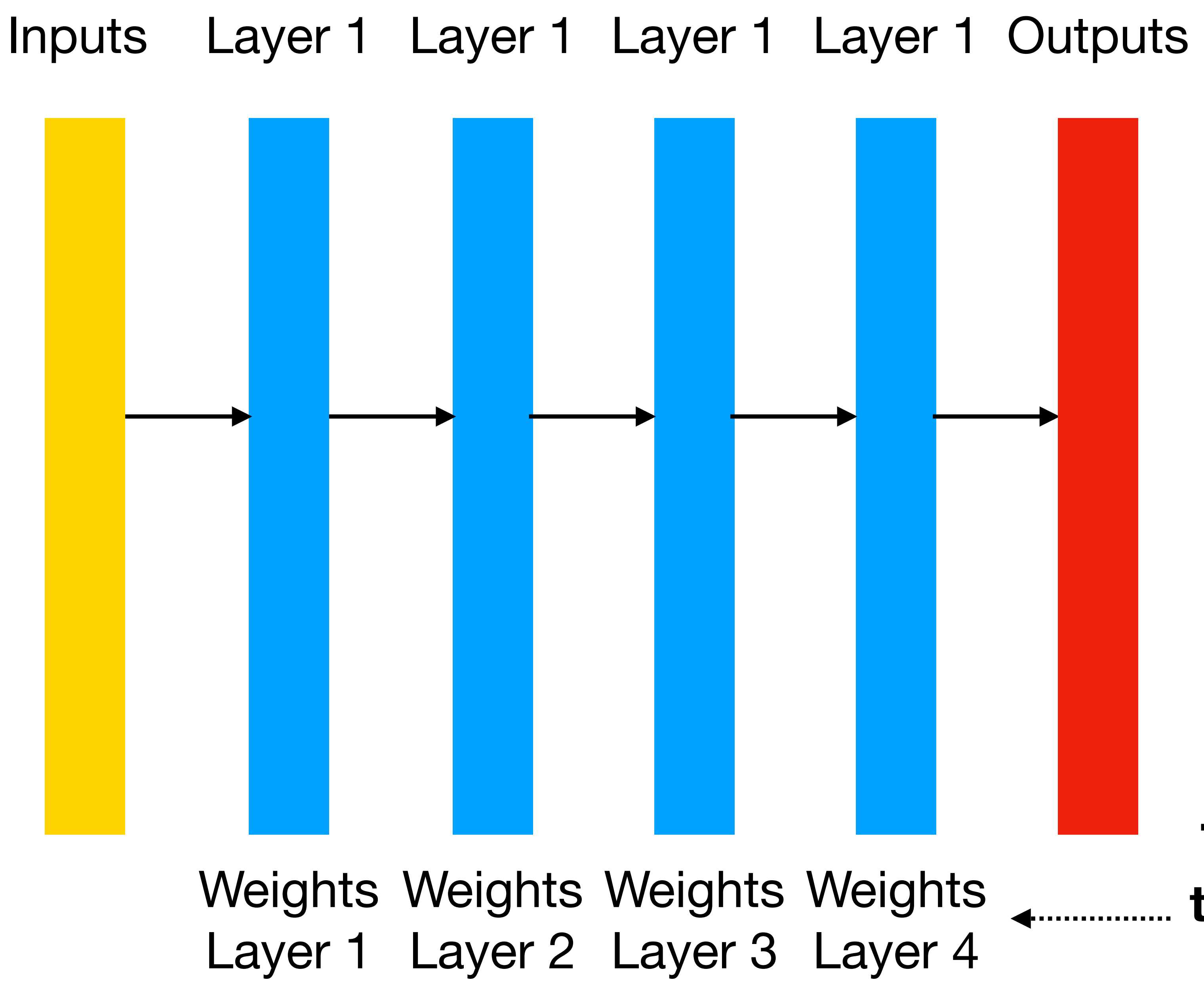
Deep Neural Networks



Deep Neural Networks

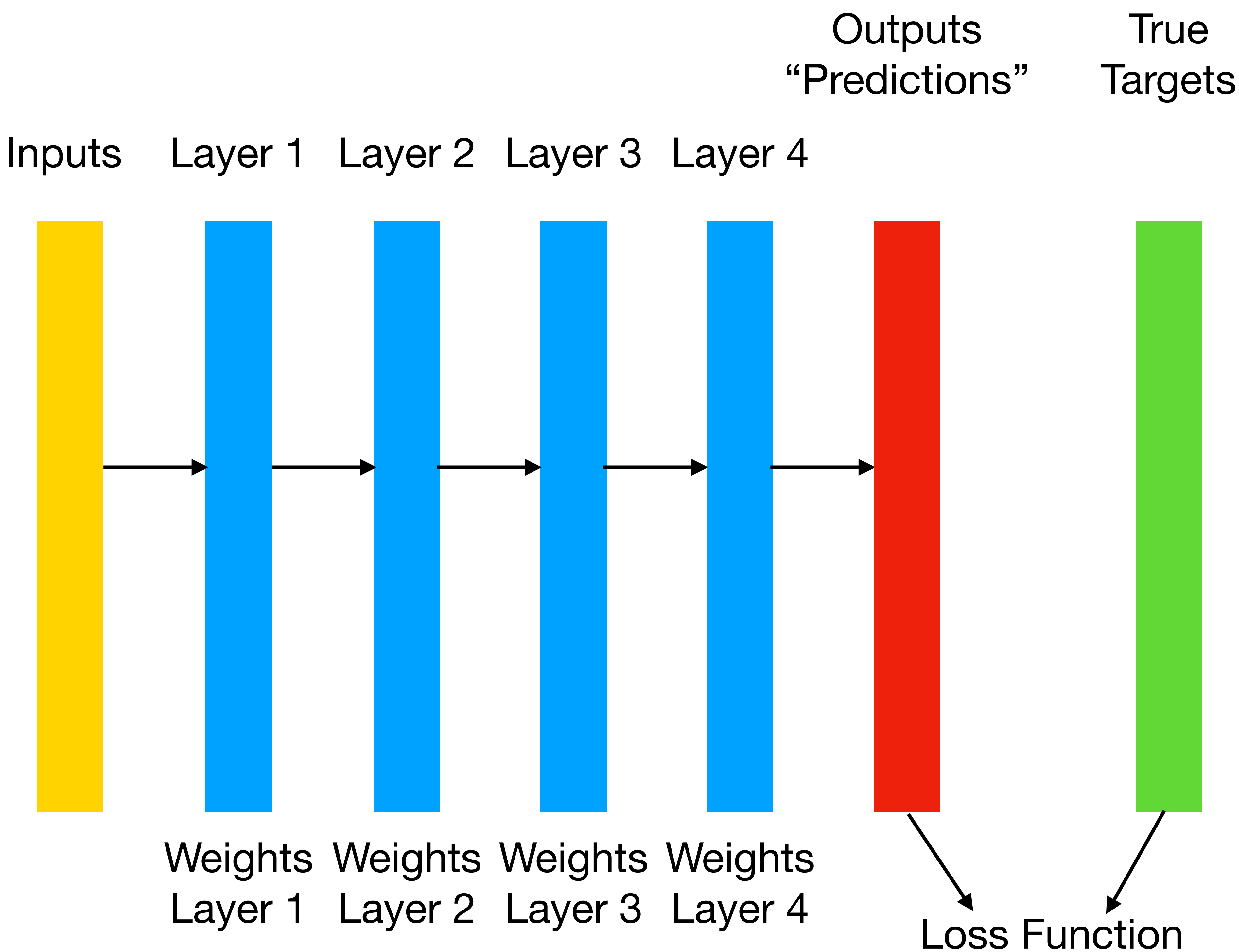


Deep Neural Networks

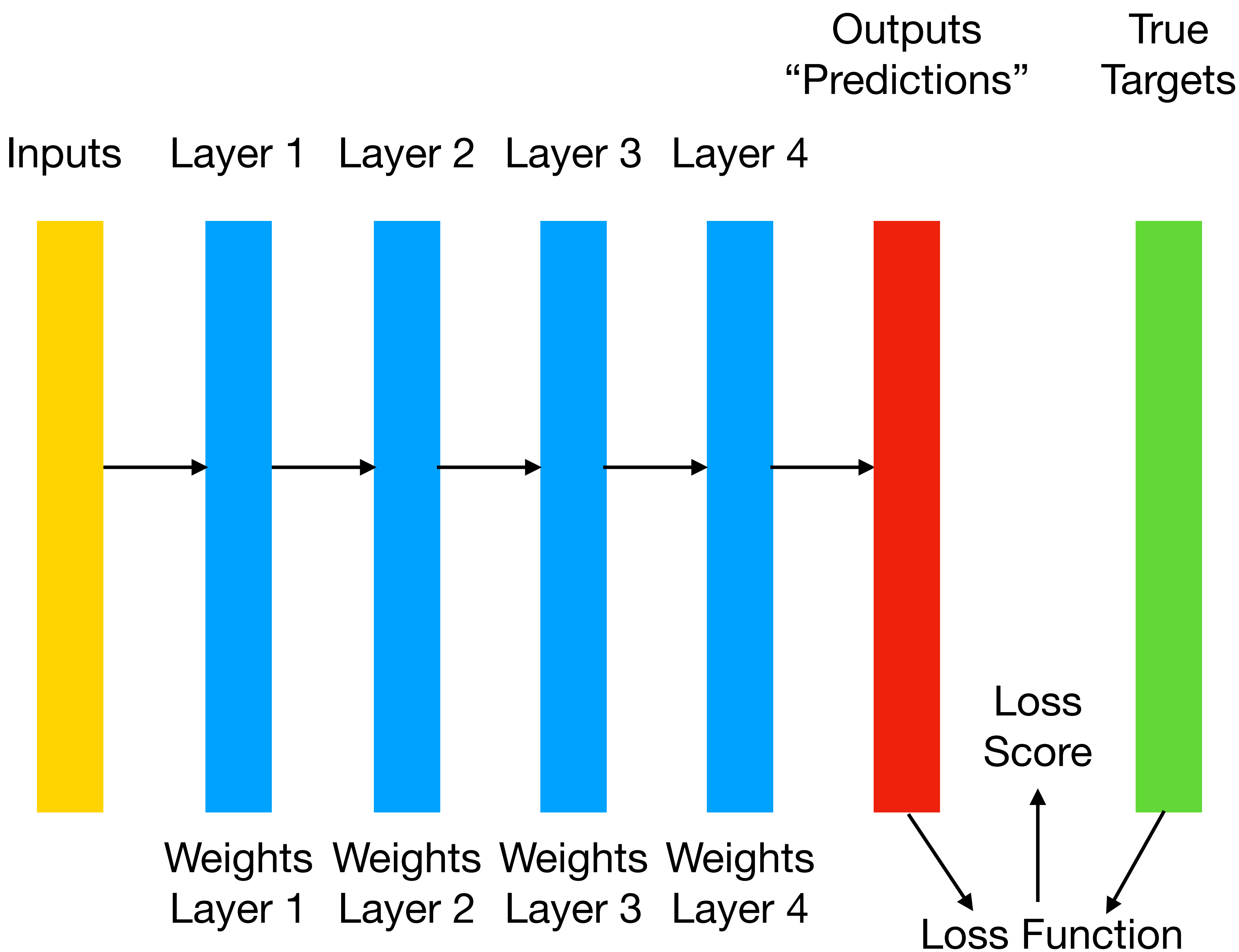


The goal is to find the right values for these weights.

Deep Neural Networks



Deep Neural Networks



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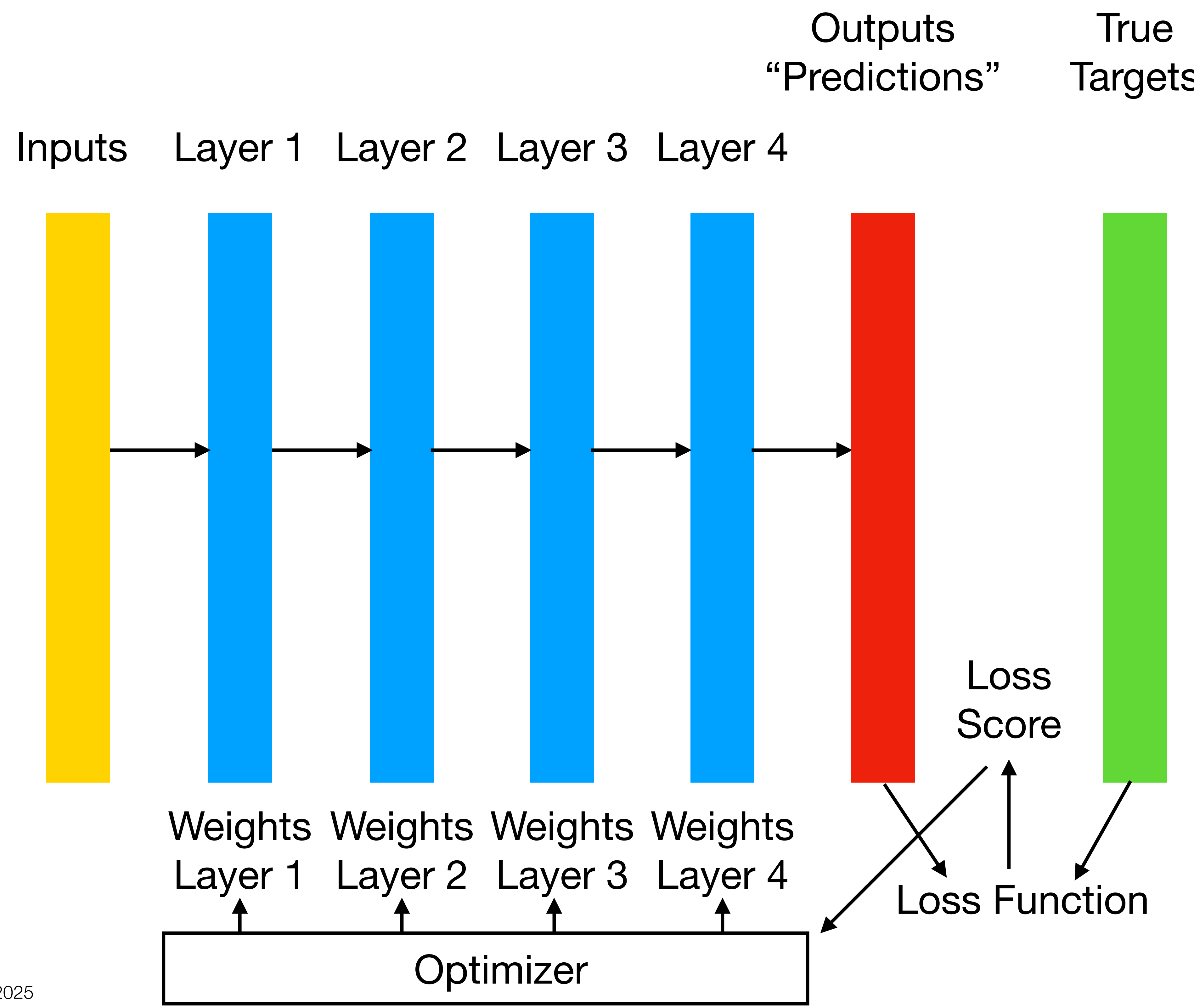

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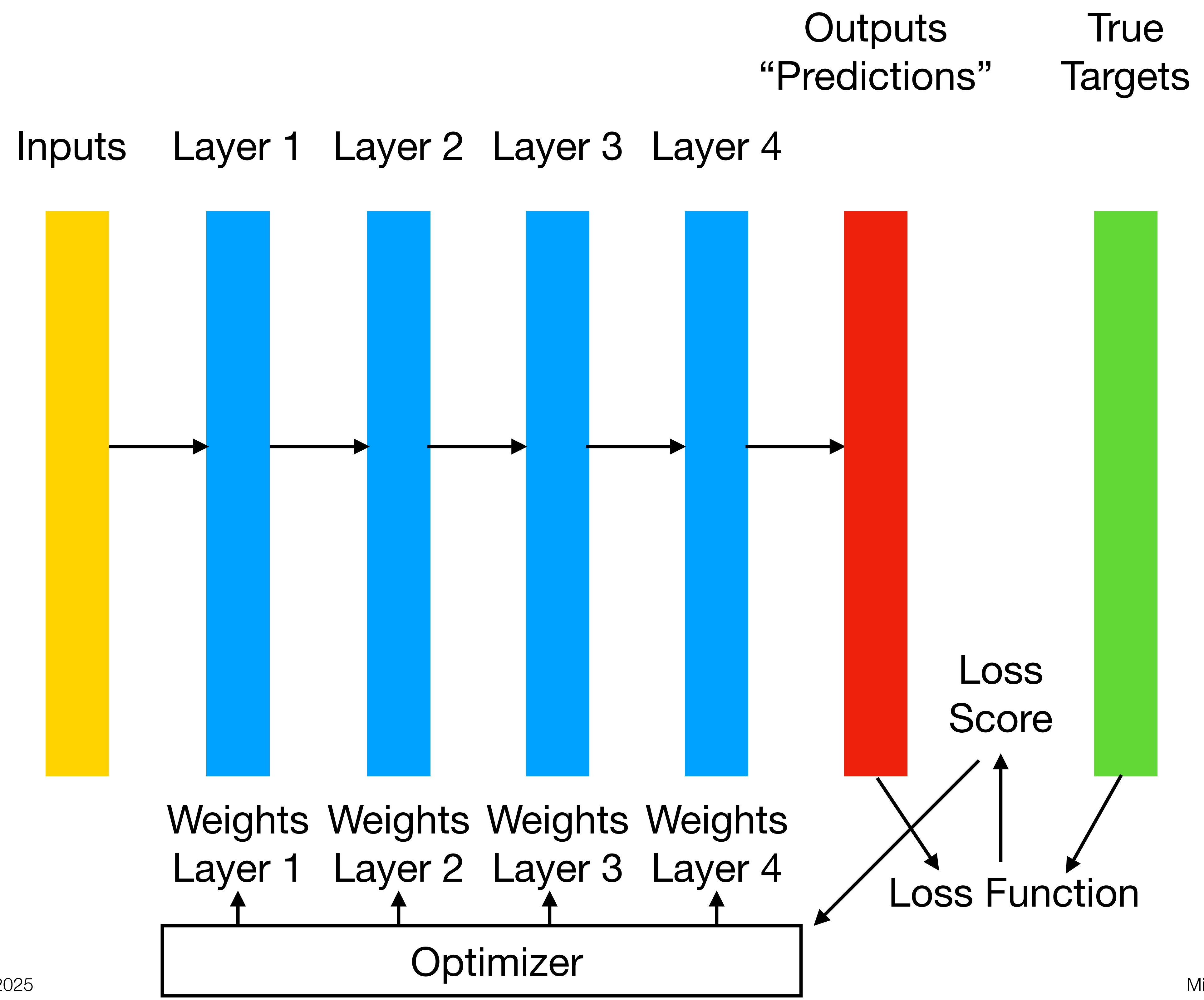
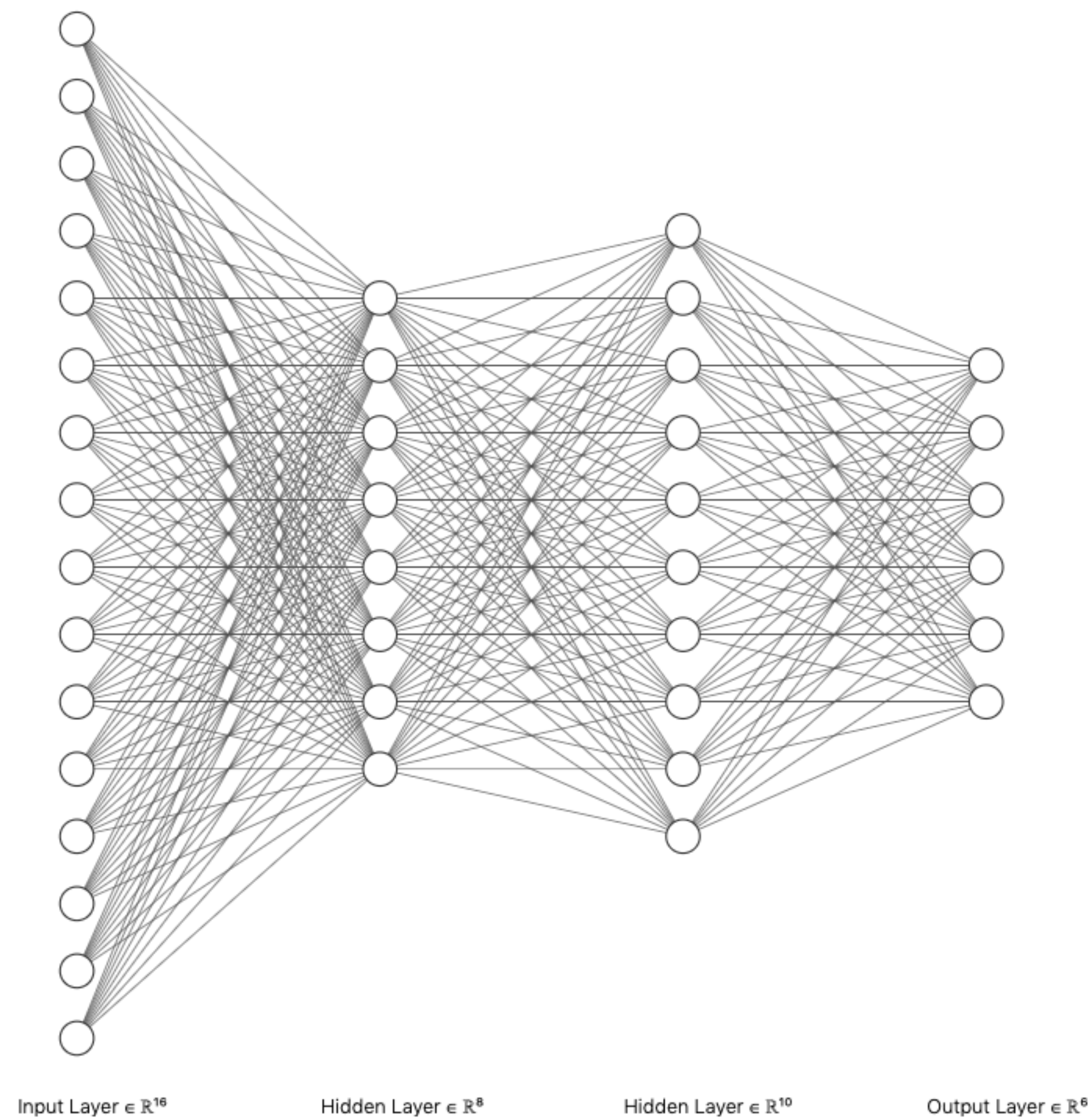
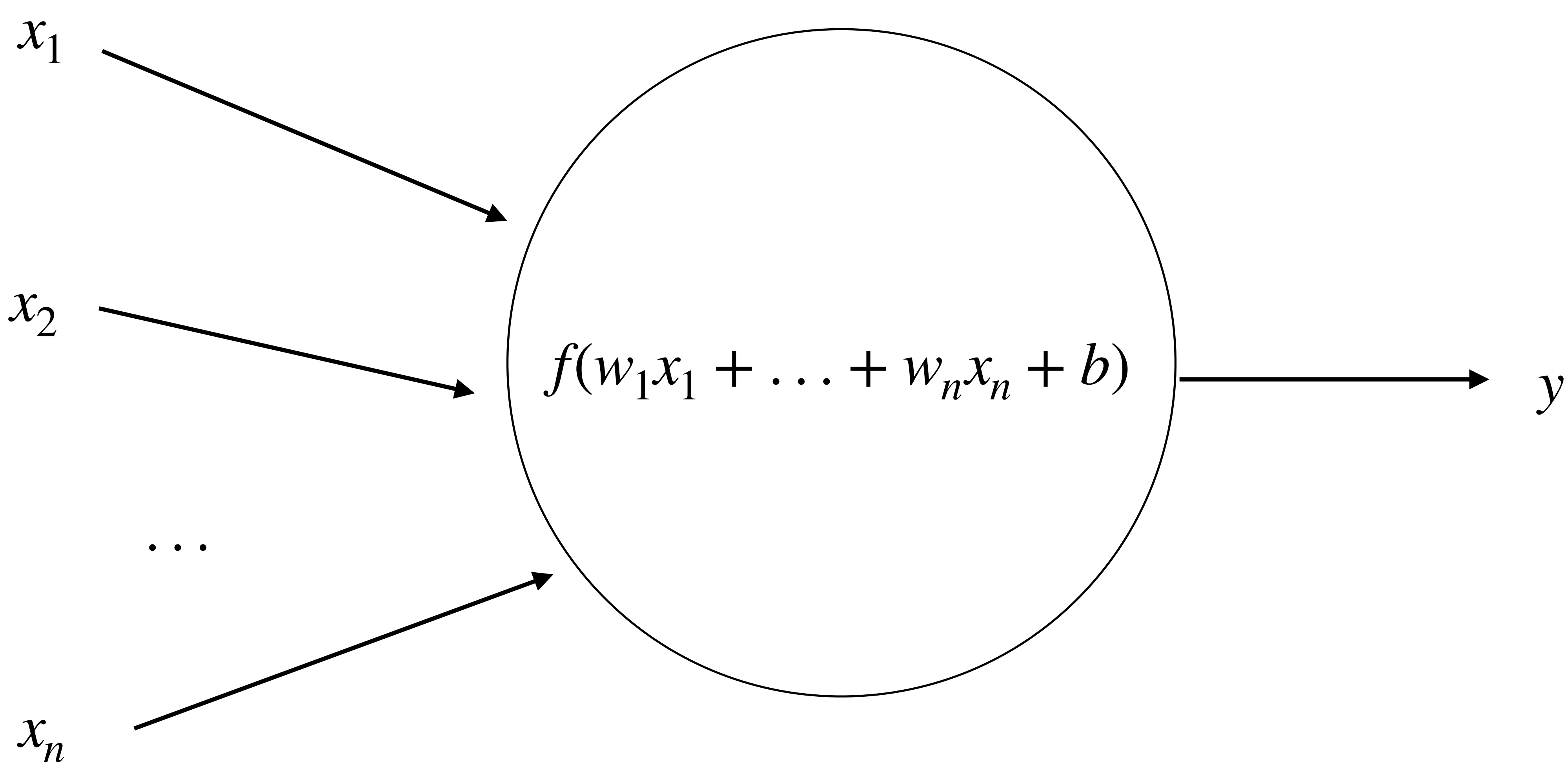


Deep Neural Networks



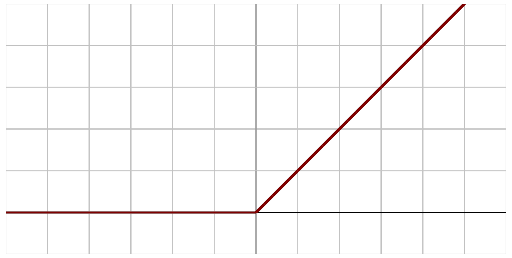
Nodes/Units/Neurons



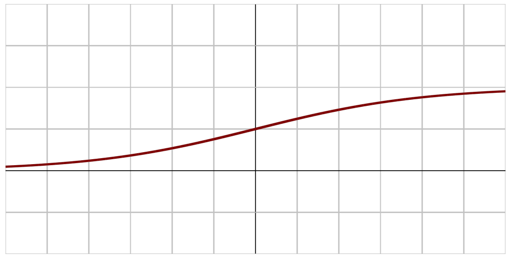
f is called the activation function, b is usually called the bias

Activations Functions

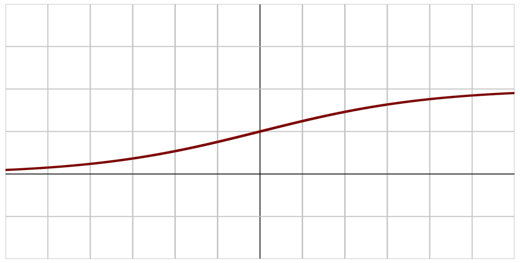
- They are generally used to add non-linearity.
- Examples:
  - *Rectified Linear Unit*: it returns the max between 0 and the value in input. In other words, given the value z in input it returns $max(0,z)$.



  - *Logistic sigmoid*: given the value in input z, it returns $\dfrac{1}{1+e^{z}}$.

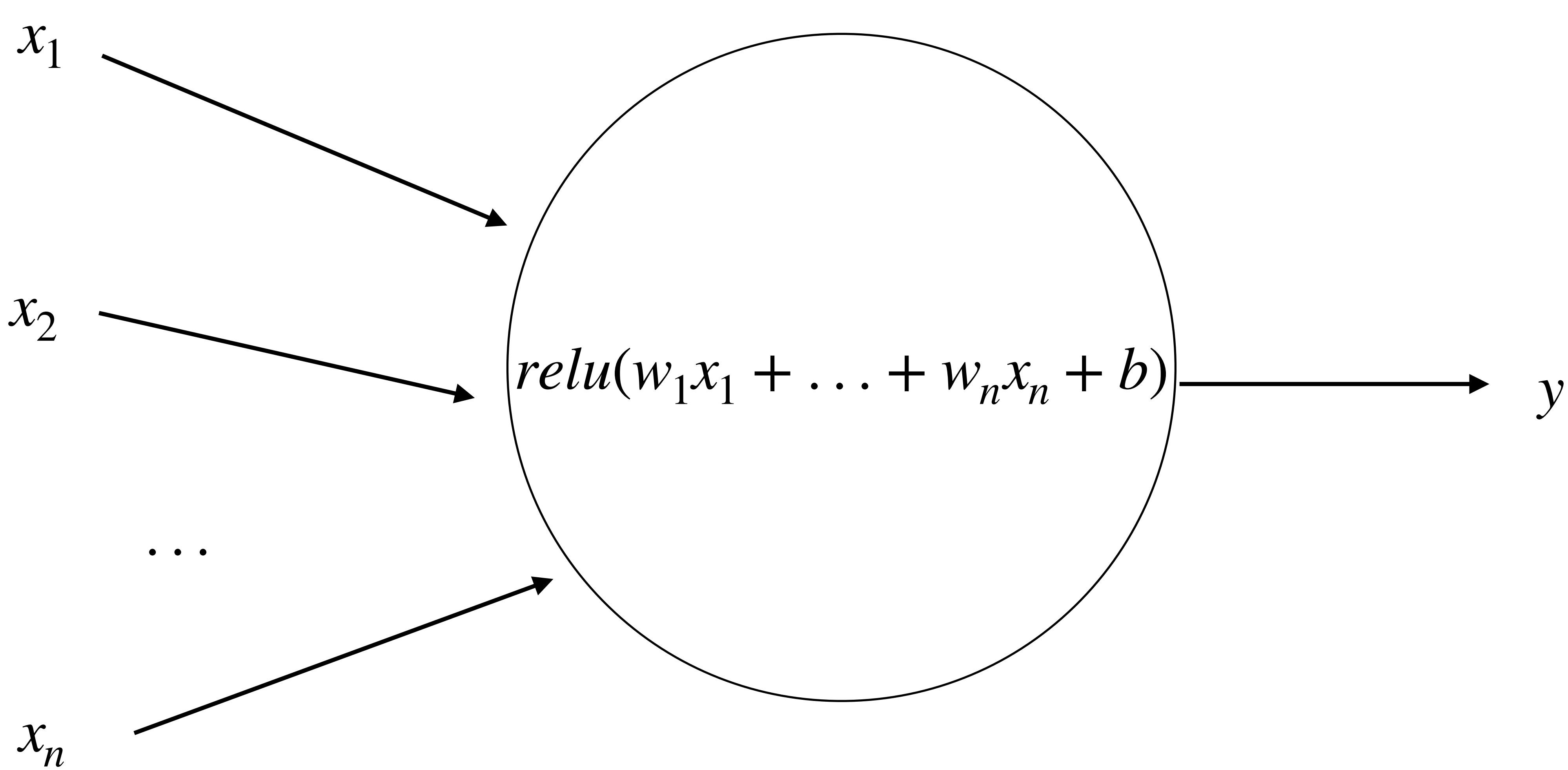


  - *Arctan*: given the value in input z, it returns $tan^{-1}(z)$.



Credit: Wikimedia

Nodes/Units/Neurons



Note that here the function in input of relu is 1-dimensional.

Softmax Function

- Another function that we will use is *softmax*.
- But please note that softmax is not like the activation functions that we discussed before. The activations functions that we discussed before take in input real numbers and returns a real number.
- A softmax function receives in input a vector of real numbers of dimension n and returns a vector of real numbers of dimension n.
- *Softmax*: given a vector of real numbers in input $\mathbf{z}$ of dimension n, it normalises it into a probability distribution consisting of n probabilities proportional to the exponentials of each element z_i of the vector $\mathbf{z}$. More formally,

$$softmax(\mathbf{z})_i = \frac{e^{z_i}}{\sum_{j=1}^{n} e^{z_j}} \text{ for } i = 1,..n.$$

Gradient-based Optimization

- We will now discuss a high-level description of the learning process of the network, usually called *gradient-based optimization*.

- Each neural layer transforms his input layer as follows:

$$output = f(w_1 x_1 + \ldots + w_n x_n + b)$$

- And in the case of a relu function, we will have

$$output = relu(w_1 x_1 + \ldots + w_n x_n + b)$$

- Note that this is a simplified notation for one layer, it should be $w_{1,i}$ for layer i.

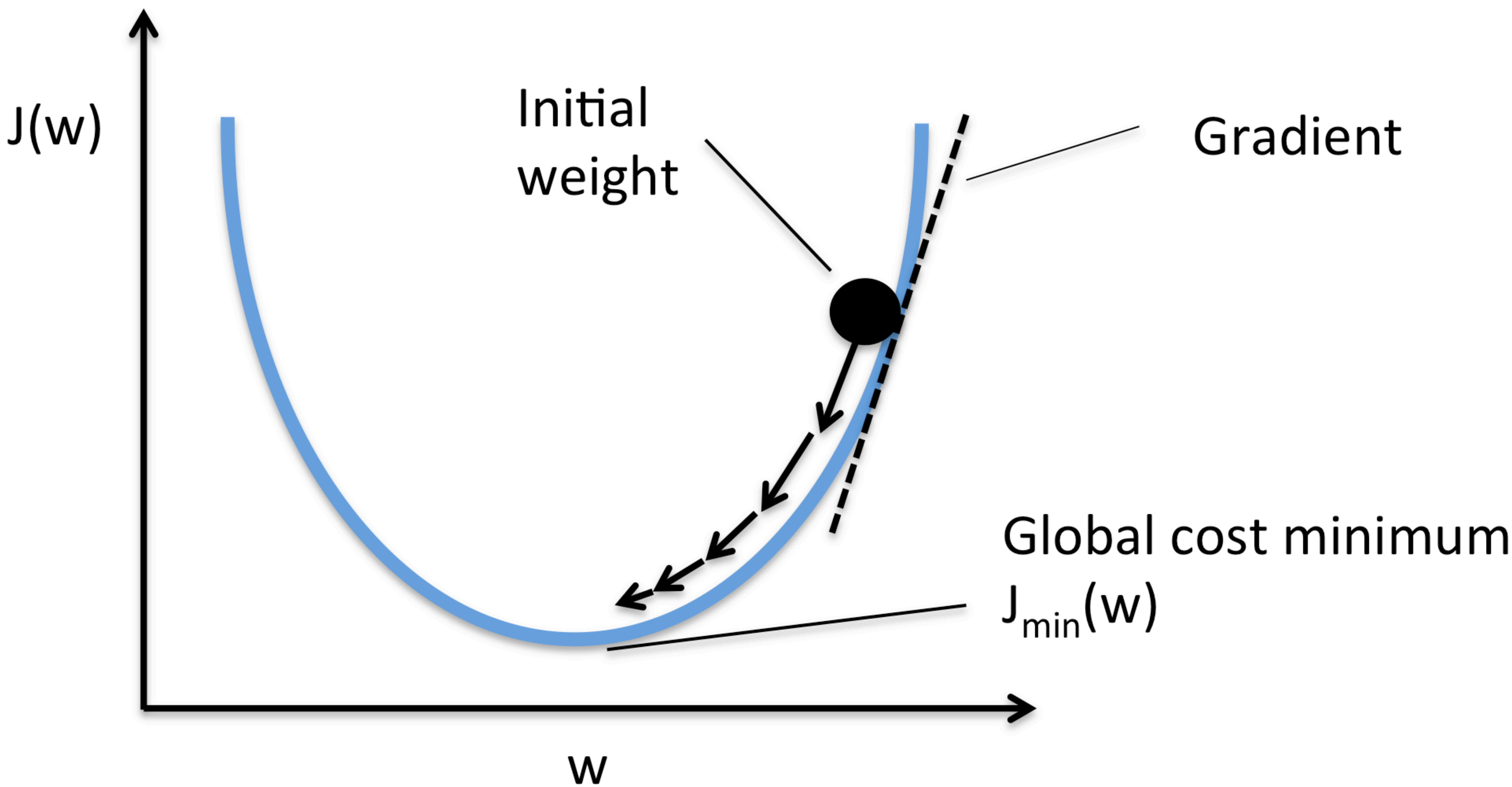
Gradient-based Optimisation

- The learning is based on the gradual adjustment of the weight based on a feedback signal, i.e., the loss described above.
- The training is based on the following training loop:
  - Draw a batch of training examples $\mathbf{x}$ and corresponding targets $\mathbf{y}_{target}$.
  - Run the network on $\mathbf{x}$ (forward pass) to obtain predictions $\mathbf{y}_{pred}$.
  - Compute the loss of the network on the batch, a measure of the mismatch between $\mathbf{y}_{pred}$ and $\mathbf{y}_{target}$.
  - Update all weights of the networks in a way that reduces the loss of this batch.

Stochastic Gradient Descent

- Given a differentiable function, it's theoretically possible to find its minimum analytically.
- However, the function is intractable for real networks. The only way is to try to approximate the weights using the procedure described above.
- More precisely, since it is a *differentiable* function, we can use the gradient, which provides an efficient way to perform the correction mention before.

Gradient-based Optimisation



Credit: Sebastian Raschka

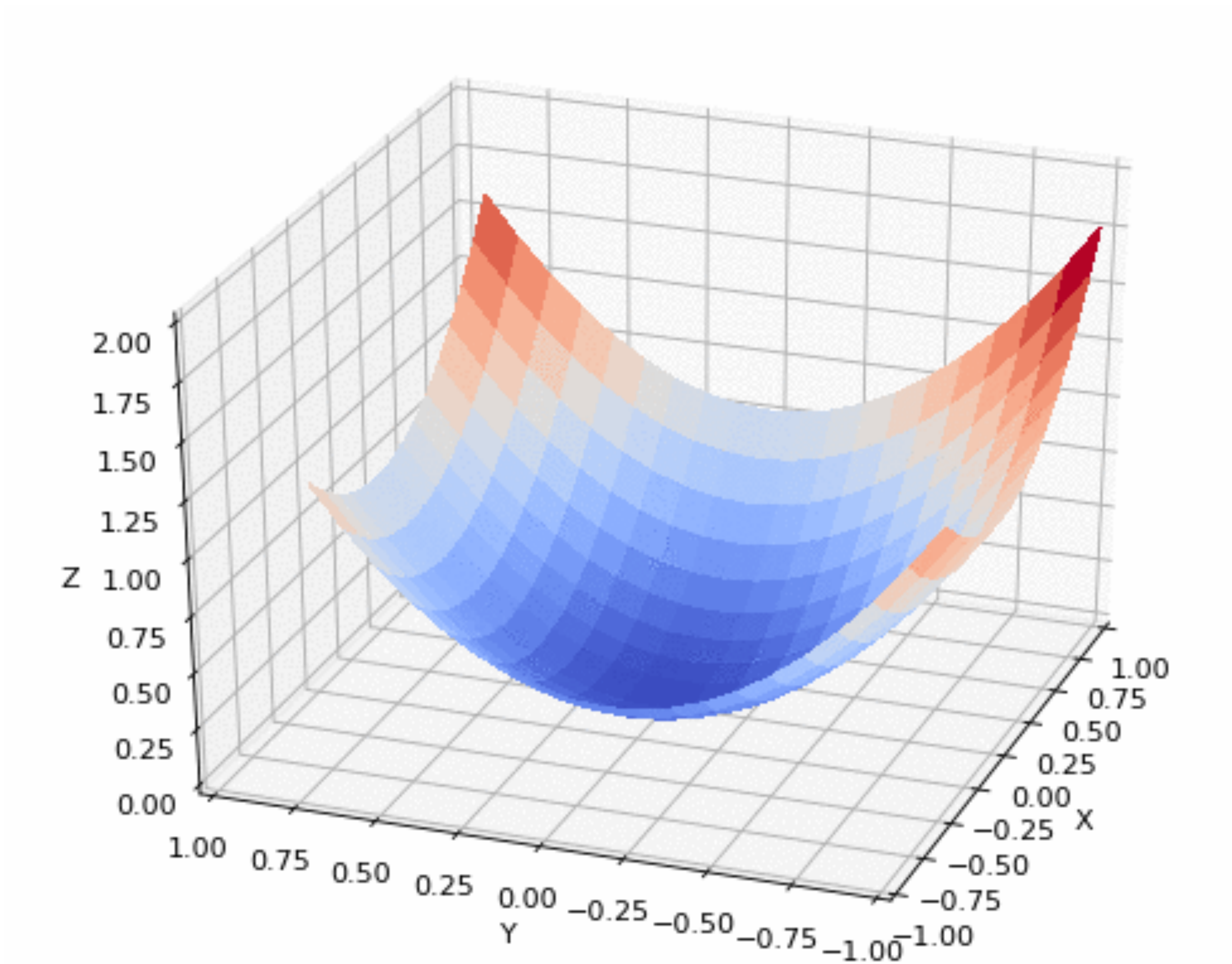
Stochastic Gradient Descent

- More formally:
  - Draw a batch of training example $\mathbf{x}$ and corresponding targets $\mathbf{y}_{target}$.
  - Run the network on $\mathbf{x}$ (forward pass) to obtain predictions $\mathbf{y}_{pred}$.
  - Compute the loss of the network on the batch, a measure of the mismatch between $\mathbf{y}_{pred}$ and $\mathbf{y}_{target}$.
  - Compute the gradient of the loss with regard to the network's parameters (backward pass).
  - Move the parameters in the opposite direction from the gradient with: $w_j \leftarrow w_j + \Delta w_j = w_j - \eta \frac{\partial J}{\partial w_j}$ where J is the loss (cost) function.
  - If you have a batch of samples of dimension k:

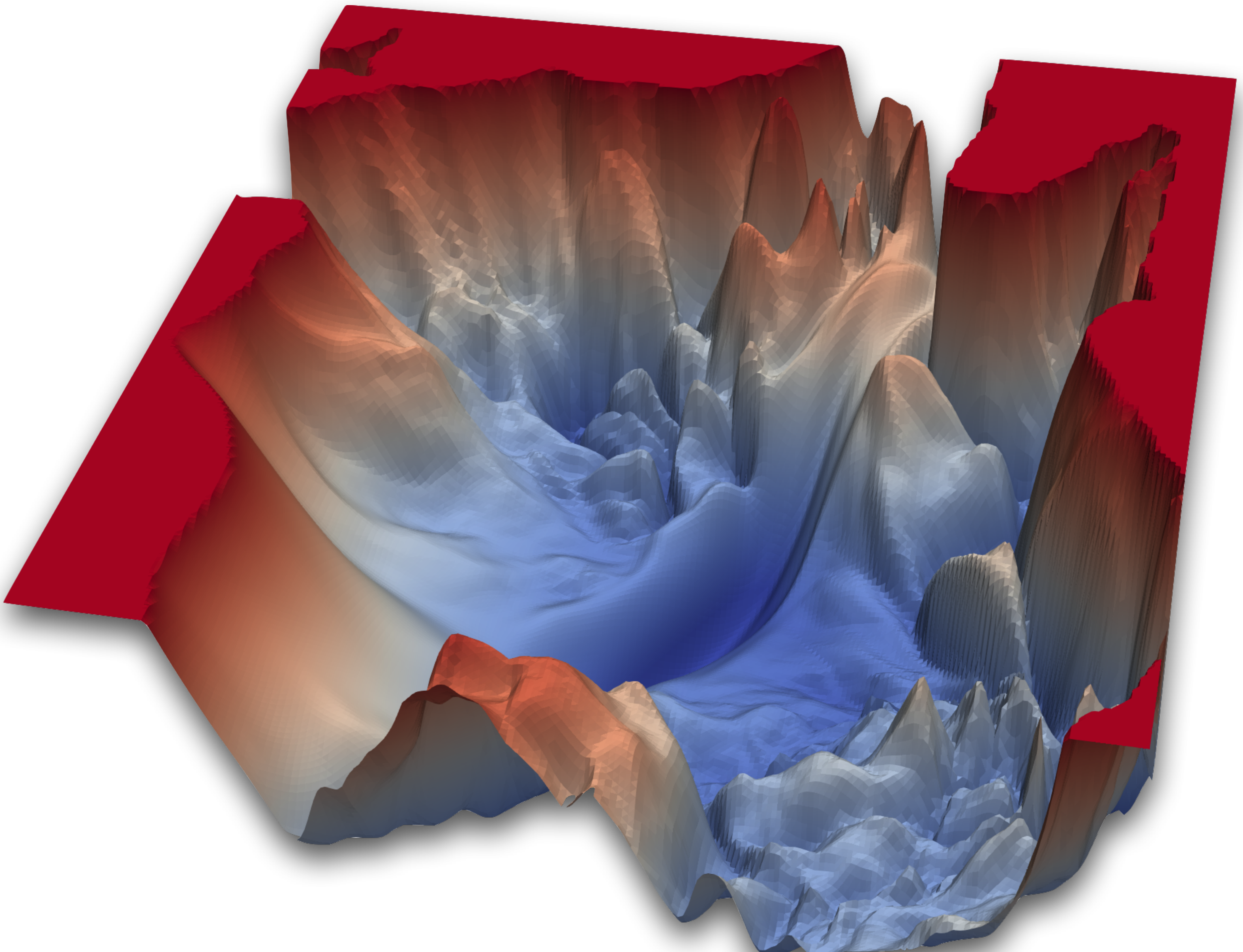
$w_j \leftarrow w_j + \Delta w_j = w_j - \eta \; average(\frac{\partial J_k}{\partial w_j})$ for all the k samples of the batch.

Stochastic Gradient Descent

- This is called the mini-batch stochastic gradient descent (mini-batch SGD).
- The loss function J is a function of $f(\mathbf{x})$, which is a function of the weights.
  - Essentially, you calculate the value $f(\mathbf{x})$, which is a function of the weights of the network.
  - Therefore, by definition, the derivative of the loss function that you are going to apply will be a function of the weights.
- The term *stochastic* refers to the fact that each batch of data is drawn randomly.
- The algorithm described above was based on a simplified model with a single function in a sense.
- You can think about a network composed of three layers, e.g., three tensor operations on the network itself.



https://blog.paperspace.com/intro-to-optimization-in-deep-learning-gradient-descent/



https://www.cs.umd.edu/~tomg/projects/landscapes/

Backpropagation Algorithm

- Suppose that you have three tensor operations/layers f, g, h with weights $\mathbf{W}^1$, $\mathbf{W}^2$ and $\mathbf{W}^3$ respectively for the first, second, third layer. You will have the following function:

$$y_{pred} = f(\mathbf{W}^1, \mathbf{W}^2, \mathbf{W}^3, \mathbf{x}) = f(\mathbf{W}^3, g(\mathbf{W}^2, h(\mathbf{W}^1, \mathbf{x})))$$

with $f()$ the *rightmost* function/layer and so on. In other words, the input layer is connected to $h()$, which is connected to $g()$, which is connected to $f()$, which returns the final result.

- A network is a sort of chain of layers. You can derive the value of the "correction" by applying the chain rule of the derivatives backwards.
  - Remember the chain rule $(f(g(x)))' = f'(g(x))g'(x)$.

Backpropagation Algorithm

- The update of the weights starts from the right-most layer *back* to the left-most layer. For this reason, this is called *backpropagation* algorithm.
- More specifically, backpropagation starts with the calculation of the gradient of final loss value and works backwards from the right-most layers to the left-most layers, applying the chain rule to compute the contribution that each weight had in the loss value.
- Nowadays, we do not calculate the partial derivates manually, but we use frameworks like TensorFlow and Pytorch that support symbolic differentiation for the calculation of the gradient.
- TensorFlow and PyTorch support the automatic updates of the weights described above.
- More theoretical details can be found in:

Ian Goodfellow, Yoshua Bengio and Aaron Courville. Deep Learning. MIT Press. 2016.

References

- Chapter 1 of Ian Goodfellow, Yoshua Bengio and Aaron Courville. Deep Learning. MIT Press. 2016.
- Chapter 2 of Francois Chollet. Deep Learning with Python. Manning 2022.